# ASTRODYNAMICS SYMPOSIUM (C1) 

Guidance, Navigation \& Control (2) (4)

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## A PAPER AND PENCIL METHOD OF EVALUATING TRAJECTORIES OF SPACE LAUNCHERS


#### Abstract

The determination of the trajectory of a space launcher is a complex optimization problem with boundary and path constraints. In particular the trajectory is initially constrained to a vertical launch, then it has to keep a zero incidence angle, the so called gravity turn phase, while several other path constraints can be active, such as: avoiding the fly over inhabited areas, controlling the fall of exhausted stages within safe regions, ensuring the launcher visibility from selected ground stations, etc. In this paper analytic formulas are developed for the gravity turn trajectory of a multistage space launcher. Formulas are given for relative velocity, flight path angle, altitude and range. All the trajectories start from the same initial condition: vertical launch with zero relative velocity and they differ by the final value of the last stage flight path angle. Coasting arches can also be included in the formulation. It is proved in the paper that from these formulas it is possible to derive: i) a rather accurate estimate of the gravitational and aerodynamic losses of a space launcher ii) sub-optimal trajectories providing approximate evaluation of launcher performances, in particular the payload mass injected into orbits of different altitude and inclination iii) determination of the pitch manoeuvre to rotate the launcher from vertical to gravity turn trajectory iv) characterization of the attitude manoeuvres that the launcher control system has to implement during the atmospheric phase of the flight

The two stage launcher Falcon 1 is considered as test case to show the effectiveness of the analytic approach proposed here.


