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FORMATION FLYING ALONG ELLIPTICAL ORBIT USING ATTRACTIVE SETS OF OPTIMAL  
CONTROL**Abstract**

The Tschauner-Hempel (TH) equations describe relative motion of a chaser satellite with respect to the target satellite in an elliptic orbit. Periodic solutions of the TH equations are used for formation flying because no control efforts are needed to maintain them. Yamanaka and Ankersen introduced the powerful state transition matrix associated with the TH equations[1]. In this solution, position and velocity of the chaser depend on an initial true anomaly  $\theta_0$  of the target. Moreover, periodic solutions are characterized by three constants.

This paper proposes a new method of optimal trajectory design for formation flying. Under linearized assumptions and a quadratic performance index, the optimal cost is quadratic in the initial state. We consider an attractive set of optimal control defined as contours of the optimal cost. The attractive set describes a set of all initial states to reach a desired state by a given cost.

In our previous research, we considered the attractive sets of optimal control for formation flying along a circular orbit. For circular orbits, relative motion of a chaser with respect to a target is described by the Clohessy-Wiltshire (CW) equations. In this case, the attractive set becomes ellipsoid and the optimal initial state for the rendezvous problem is uniquely determined as the point where the ellipsoid is tangent to the chaser's initial orbit. We revealed the relation between the properties of the attractive set and the optimal initial state of the chaser in connection with the weight parameters in the performance index.

In this paper, we extend the idea of the attractive set of optimal control to an elliptical orbit, where the linearized motion is time periodic. By solving the optimal control problem for TH equations, the optimal cost is obtained as a time-periodic function. Therefore, the attractive set defined by the optimal cost also becomes time-periodic. First, we demonstrate how the shape of the periodic orbit and the attractive sets change by varying initial true anomaly of the target satellite and size and phase parameters of periodic orbits. Then, it is found that by changing the initial phase of the periodic orbit, we can determine the optimal initial state uniquely. Finally, the effects of eccentricity and flight time on the shape of the attractive sets are discussed.

[1] Yamanaka, K., and Ankersen, F., Journal of Guidance, Control, and Dynamics, Vol. 25, No. 1, 2002, pp. 60 – 66.