ASTRODYNAMICS SYMPOSIUM (C1)
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# ON THE CALCULUS OF VARIATIONS IN THE OPTIMAL SPACECRAFT FORMATION FLYING PATH PLANNING

**Abstract**

The path planning problem for spacecraft formations, assigned the mission scenario and the mission objectives, requires optimizing the performances of the formation in terms of given indices. Typical performance indices are coverage, time, robustness, or control effort. The problem of accomplishing a given mission while minimizing the *control effort* is herein solved.

Optimality of results achieved is guaranteed by *necessary* and *sufficient* conditions of Calculus of Variations, the branch of mathematics aimed at finding extremals of functionals. In addition, results achieved ensure both compactness to the formation and collision avoidance among its elements.

Spacecraft are modeled as 6 DOF rigid bodies subject to radial gravitational forces under Keplerian assumptions, perturbing forces due to the zonal coefficient $J_2$, perturbing forces and torques due molecular air impingement, and finally forces and torques provided by the control subsystem.

The control effort for the $i$-th element of the formation is quantified in terms of fuel consumption and energy consumption, which are the integral of the 2-norm and the integral of the square of the 2-norm of the control accelerations respectively.

The scope of the tackled problem is to find the optimal trajectory in terms of position and attitude vectors such that the assigned cost indices are minimized over a given time interval and such that vehicles maintain a specified distance from each other.

Effectiveness of results achieved is shown by numerical simulations in which each element of a formation has to accomplish these tasks: 1. leave from an unspecified initial position at unspecified velocity and reach an assigned waypoint, unique for the entire formation, 2. pass through $m$ assigned waypoints at prescribed velocity, 3. avoid a no-fly zone modeled a sphere, 4. reach a given regular surface.

The approach employed ensures the *modularity* of the solutions provided: the phases of the mission considered can be effortlessly rearranged or varied by tuning a finite amount of parameters.

This work is part of a systematic analytical study aimed at solving the formation flying problem and capitalizes on previous results achieved by the authors employing Calculus of Variations and modern Optimal Control Theory.