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NEW NUMERICAL METHODS FOR DETERMINING PERIODIC ORBITS IN THE CIRCULAR RESTRICTED THREE-BODY PROBLEM

Abstract

The generation of periodic orbits in the context of the circular restricted three-body problem has been pursued with different approaches, for decades. The equations of motion for the problem at hand are not amenable to an analytical solution, and therefore past studies are mostly focused on analytical approximations, as those provided by Richardson, Gomez, and Mancote. Shooting algorithms and generating functions represent alternative numerical techniques employed in the past, but they exhibit limitations due to the high sensitivity to the starting guess and to the reduced spatial range of applicability, respectively. This paper formulates the problem of determining periodic orbits as an optimal control problem, and applies two distinct methods to its solution. Position and velocity define the dynamical state of the third body and, by definition, for a periodic orbit they must match with the respective initial values after a single period. This means that a periodic orbit is such that the initial and the terminal values of the state variables coincide. The objective function adopted in this work is the magnitude of the vector that represents the displacement between the initial state and the terminal state. A periodic orbit minimizes this objective function to 0. The first method is represented by the direct collocation with nonlinear programming (DCNLP) algorithm, which is based on the conversion of the original problem into a nonlinear programming problem, through the discretization of the state variables and the translation of the equations of motion into nonlinear equality constraints. In many applications the method at hand turns out to be fairly robust even in the presence of poor initial (guess) solutions. This paper proves that the first order Richardson's analytical expansion is sufficient (as a starting guess) to ensure convergence of the DCNLP algorithm, which successfully finds Lyapunov and Halo orbits. The second method is the particle swarm optimization technique, which is a stochastic, population-based methodology inspired by the unpredictable motion of bird flocks. Unlike the DCNLP method, the particle swarm algorithm does not need any guess solution, and successfully finds Lyapunov and Halo orbits with great accuracy. However, equality constraint handling is more challenging when stochastic methods are employed. This paper shows how the equations of motion can be rewritten in order to avoid considering the Jacobi integral as an equality constraint. This research describes these two novel approaches to determining periodic orbits, their respective favorable features, and their drawbacks.