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DIRECT TRANSCRIPTION OF LOW-THRUST TRAJECTORIES WITH FINITE TRAJECTORY  
ELEMENTS

**Abstract**

The design of a low-thrust trajectory requires the definition of the thrust profile that satisfies a two-point boundary value problem. The scope of this work is to provide a computationally efficient way to determine a good approximated solution to this problem with a more accurate representation of the control profile compared to other approximation methods. In the literature the problem has been tackled in different ways generally classified in two families: indirect methods and direct methods. Indirect methods translate the design of a low-thrust trajectory into the solution of an optimal control problem and derive explicitly the associated first order optimality conditions. The first order optimality conditions are a system of mixed differential-algebraic equations (DAE). Multiple-shooting, collocation and approximated analytical approaches have been proposed to solve the DAE system and satisfy the boundary conditions. Direct methods, do not derive the optimality conditions but transcribe the equations of motion into a system of algebraic equations and then solve a nonlinear programming problem. Numerical integration and collocation techniques have been proposed to transcribe the differential equations. Both direct and indirect methods require some form of first guess solution. In the past decade, some low-fidelity approximation techniques have been proposed to generate the first guess solution. However, the use of these low-fidelity solutions is not always straightforward. More recently a fast direct method based on the transcription of a low-thrust trajectory into a multi-burn transfer has been proposed to generate a medium-fidelity solution at a low computational cost. In this paper, a direct method is presented where the trajectory is decomposed into a number of finite elements. Gauss planetary equations are then solved over each element by means of a perturbative approach, for constant thrust modulus and direction. The trajectory is assumed to be an epsilon-variation of a Keplerian arc, where epsilon is a ‘small’ acceleration term due to the low-thrust. A very fast transcription of the trajectory into a nonlinear programming problem is thus obtained, the accuracy of which is controlled by the number of elements, assuming that every trajectory element remains a first order epsilon-variation of a Keplerian arc. We will show how this approach can be used for fast multiobjective optimization of long low-thrust spirals where both the mass of propellant and the transfer time need to be minimized.