

IAF ASTRODYNAMICS SYMPOSIUM (C1)
Mission Design, Operations & Optimization (2) (7)

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APPROXIMATE CLOSED-FORM SOLUTION OF THE PERIODICALLY PERTURBED TWO-BODY
PROBLEM WITH APPLICATION TO LOW-THRUST OPTIMIZATION**Abstract**

Approximate analytical expressions for the orbital evolution enable one to calculate spacecraft trajectories without numerically integrating the equations of motion. Such a solution can be exploited as an initial guess in an optimization method, thus speeding up the convergence process. It is especially useful in optimization of many-revolution low-thrust maneuvers, when an optimal transfer trajectory is difficult to determine. One way to approximate the spacecraft motion is by using averaging methods from the general perturbation theory. The averaged equations of perturbed spacecraft motion define the secular evolution of orbital elements, with their short-period oscillations excluded. However, standard averaging approaches do not lead to the equations with a closed-form solution.

In this work, the authors present a novel approximate analytical model for spacecraft orbital motion in the perturbed two-body problem. The equations of motion are written in terms of modified equinoctial elements and averaged over the true longitude under the assumption that the perturbing acceleration is represented by the Fourier series in true longitude with constant or piecewise constant coefficients. The averaged system of differential equations is found to have a closed-form solution depending on only eight scalar parameters of the perturbation. The derived solution has certain limits of applicability. First, the perturbing acceleration should be small compared to the central gravitational acceleration, which is true for a broad range of natural perturbations and also for the low-thrust control acceleration. Second, orbits of low to moderate eccentricity are assumed: the higher the orbital eccentricity, the lower the accuracy of the proposed averaging scheme.

The purpose of this report is to demonstrate how the derived approximate solution is used to solve optimal control problems. Since the averaged equations contain only a few Fourier coefficients of the perturbing acceleration spectrum, the control function can be parameterized by a few variables. The optimal control problem is thus reduced to a nonlinear programming problem of low dimension. The energy-optimal control law is obtained for the many-revolution low-thrust transfer problems of orbit raising and maneuvering between Keplerian orbits. The usage of the closed-form solution reduces the computational time to the time spent on a single numerical propagation of the trajectory, which can obviously be performed on board. Calculated optimal trajectories are compared with solutions of the non-averaged equations of motion with the same control, and the accuracy of trajectory approximation is found to decrease with increasing the orbital eccentricity and the maneuver duration.