

ASTRODYNAMICS SYMPOSIUM (C1)  
Guidance, Navigation and Control - Part 3 (9)

Author: Dr. Mohammad Ebrahimi  
Aerospace Research Institute, Iran, mebrahimi@ari.ac.ir

Mr. Ehsan Amani  
Aerospace Research Institute, Ministry of Science, Research and Technology, Iran, amani@ari.ac.ir  
Prof. Jafar Roshanian  
K. N. Toosi University of Technology, Iran, roshanian@kntu.ac.ir

DERIVATION OF A COMPLETE SET OF EQUATIONS OF MOTION FOR COUPLED  
SLOSH-VEHICLE DYNAMICS

**Abstract**

Equations governing motion of space vehicles must address several subsystems. They include dynamics, control, and guidance subsystems. Dynamics subsystem itself is governed by rigid body, elasticity, and slosh dynamics equations. Dynamics equations have been derived by many researchers using different levels of simplifications. For example, equations of motion for elastic vehicles in 6-DoF flight were derived earlier neglecting the effect of propellant sloshing. Coupled slosh-rigid body dynamics in planar flight was investigated too, but effect of elasticity was not incorporated and extension of equations to 6-DoF flight is not straightforward as well. In current study, a new contribution in deriving dynamics equations for 6-DoF flight is made by considering three effects of slosh, elasticity, and rigid body dynamics altogether. Classically, liquid sloshing is modeled by replacing liquid in each tank with a fixed and several oscillatory masses representing slosh modes. In our work, slosh dynamics was modeled by a series of spherical pendulums instead of simple pendulums which are valid just in planar motion and small amplitude slosh oscillations (linear slosh). Equations are derived in body frame using two methods. First, we used Lagrange equation in inertial frame and transformed resultant equations into body frame. Alternatively, Lagrange equation in terms of quasi-coordinates, also called Boltzmann-Hamel equation, was utilized and same set of equations were obtained in both cases. In next stage, as a validation of our results, we simplified our equations with the assumptions used in previous works and obtained the same set of equations available in literature. The derived set of equations is capable of modeling non-linear slosh dynamics and also non-linear interactions between all dynamics subsystems in a 6-DoF vehicle flight.