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COMPUTATION OF WEAKLY STABLE STRUCTURES IN THE RESTRICTED THREE-BODY  
PROBLEM USING DIFFERENTIAL ALGEBRA TECHNIQUES**Abstract**

Stable sets are particular regions defined in the restricted three-body problem where the motion of the massless particle is confined about the smaller primary. They are computed on the basis of a simple geometrical definition of stability, which easily extends to more complex models, such as the full n-body dynamics. Forward and backward stable sets can be manipulated in order to compute ballistic capture orbits of practical interest.

In previous works, stable sets have been constructed by point-wise sampling in both the circular and elliptic restricted three-body models. In this approach, a large number of initial conditions is propagated, yielding a grid of weakly stable and unstable points. This method directly exhibits possible post-capture osculating orbital parameters, but exposes little qualitative information on the dynamics.

Differential algebra techniques allow the expansion of the flow of a dynamical system as a polynomial in terms of the initial condition. Combined with automatic domain splitting this allows the fast, accurate and efficient propagation of large sets of initial conditions. In the process, the initial condition is automatically subdivided as required by the dynamics to ensure convergence of the resulting polynomial expansions. This process not only yields a set of polynomial expansions of the final state, but also the structure of the splitting of the initial condition which contains additional information about the strength of the non-linearities in different regions of phase space.

Utilizing differential algebra to propagate the initial phase space in the three-body problem allows a more efficient computation of weakly stable structures. The resulting splitting structure provides additional insights into the dynamics of the system, and is closely related to the structure of the invariant manifolds (in the circular problem) and the Lagrangian Coherent Structures (in the elliptic problem). From a practical point of view, differential algebra also allows sensitivity analysis to be performed on system parameters to ascertain the robustness of the developed transfers.

In this paper we elaborate on the application of the differential algebra techniques for the computation of the weakly stable structures. In particular, we show that using differential algebra can be more efficient than sampling the search space, and yields additional information that can be exploited to understand the free transport in these regions. Examples are shown for the circular (Sun-Jupiter) and the elliptic (Sun-Mercury/Mars) three-body problems.