ASTRODYNAMICS SYMPOSIUM (C1)
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THE CONSEQUENCES OF STIFFNESS IN NUMERICAL SOLUTIONS OF ASTRODYNAMICS PROBLEMS

Abstract

This paper confirms the stiff nature of astrodynamics equations and examines the consequences of failing to recognize the stiff numerical phenomena. It extends the concepts of our last report to the IAC, "Analysis of the Suitability of Analytical, Semi-Analytical, and Numerical Approaches for Important Orbit Propagation Tasks." We demonstrated that the equations in any form, whether full dynamics or semi-analytic, are stiff by conventional measures. We will also extend stability and stiffness examination to Lyapunov exponents. The tests reveal relationships between temporal and spatial integration intervals. Others have shown that an incorrect relationship between time and space intervals produce realistic but very wrong outcomes that are potentially unstable. We use near equatorial, near circular orbits to illustrate the phenomena. The equatorial orbit perturbation problem is Chapter 28 in Multiple Scales Theory and Aerospace Applications by Rudrapatna V. Ramnath. The "full dynamics" equations include the J2 term (oblateness) of the geopotential. The equations are stiff. The chapter introduces an asymptotic expansion in the oblateness parameter, leading to an analytical expression for asymptotic behavior. There is also an exact solution in Elliptic Integrals. These can be a benchmark for different numerical approaches. The question is whether the choices of integration time intervals for stability are more suitable for long term propagation than those for "full dynamics," and how well each compares to an exact or asymptotically analytical solution. Everything is approximate, since the numerical integration suffers round off and other errors in approximating standard functions. It is reasonable to expect that propagating an orbit for years and years almost always misses the "real" answer by a bunch. This is very important for estimating orbit lifetime. Governing phenomena span many diverse time scales. The atmosphere suffers noteworthy changes on scales from hours to decades (solar "cycles"). The changes are marginally predictable. Standard lifetime estimation techniques consider orbital energy dissipated by drag until apogee is smaller than a fixed altitude. They do not integrate equations of motion. We will address these issues in a situation with exact, approximate, and numerical solutions. We will demonstrate that there are numerical interval boundaries beyond which thee are reasonable, but very incorrect, integrations. These boundaries depend on the independent parameters of the problem, such as time and length scales of external phenomena.