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PERIODIC ORBITS DESIGN BASED ON THE CENTER MANIFOLD THEORY IN THE CIRCULAR RESTRICTED THREE-BODY PROBLEM

Abstract

Invariant manifolds such as stable and unstable manifolds are often used in trajectory designs in the circular restricted three-body problem (CRTBP). Focusing on center manifolds, this paper proposes a simple but powerful trajectory design method for a periodic orbit based on the center manifold theorem.

In general, there exist three types of invariant manifolds around equilibrium points: stable, unstable and center manifolds. The trajectories on center manifolds are bounded, whereas those on stable or unstable manifolds exponentially approach or diverge from the libration point as time passes. Hence, periodic orbits can be considered as systems lying on center manifolds.

A general system around an equilibrium point is given by

$$\begin{aligned} \dot{x} &= Cx + f(x, y, z) \\ \dot{y} &= Py + g(x, y, z) \\ \dot{z} &= Qz + h(x, y, z) \end{aligned} \tag{1}$$

where f, g and h are nonlinear terms and all real-parts of eigenvalues of matrices C, P and Q are zero, negative and positive, respectively. According to the center manifold theorem, in the neighborhood of the equilibrium point, the system (1) can be represented by

$$\begin{aligned} \dot{x} &= Cx + f(x, \phi(x), \psi(x)) \\ y &= \phi(x) \\ z &= \psi(x) \end{aligned} \tag{2}$$

The first ordinary differential equation is called reduced order system. The solution to Eq. (2) represents solutions restricted on center manifolds.

Based on Eq. (2), we propose a novel method to obtain periodic orbit. It includes two steps. The first step computes approximate solutions for the system shown as Eq. (2) in the CRTBP. This step requires iteration by using a successive approximation method proposed by Suzuki et al., which requires initial values of the reduced order system, $\xi \in R^4$. Note that since in the first step quasi-periodic orbits are obtained, a further modification of ξ is necessary to obtain pure periodic orbits. The second step corrects ξ to form a periodic orbit. The required variation $\Delta\xi$ are computed by using the state transition matrix.

The remarkable feature of the proposed method is that once we fix three components of ξ, the method provides the initial state for a periodic orbit uniquely by successive iteration mentioned above. Note that two of them become zero from symmetries of periodic orbits and the rest can be set arbitrarily.

Contrary to the proposed method, conventional methods such as the differential correction method and the Lindstedt-Poincaré method require good initial guesses and complex algebraic manipulations.

The proposed method is applicable to any equilibrium point, although as an example the proposed method is applied to the Sun-Earth L_2 point and verified in numerical simulation. In conclusion, we reveal that the proposed method is a very powerful tool and can significantly reduce efforts to obtain periodic orbits.