Paper ID: 35856
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ASTRODYNAMICS SYMPOSIUM (C1)
Mission Design, Operations & Optimization (2) (5)

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BREAKWELL LECTURE: INVARIANT MANIFOLDS IN ASTRODYNAMICS

Abstract

One of the key points to understand the behaviour of a dynamical system (a system that evolves with time) is the study of its invariant manifolds. This was already shown by J.H. Poincaré that showed that the three-body problem is not completely integrable, in the sense that, besides the energy, and the angular and linear momenta, there are no other analytical first integrals of the system which commute with the energy. His proof was based on the existence of what nowadays is called a "homoclinic tangle," this is the infinite transverse crossings of the stable and unstable manifolds (asymptotic and doubly asymptotic solutions) of a certain orbit that accumulate at the same orbit. The existence of this kind of complicated sets guarantees also the existence of random (chaotic) motions of the system. In connection with the homoclinic tangle Poincaré wrote: One will be struck by the complexity of this figure, which I do not even try to draw. Nothing is more likely to give us an idea of the complexity of the Three-Body Problem and in general of all the problems of dynamics where there is no uniform integral and where the Bohlin series are divergent. (LMNMC, Chapter XXXIII, section 397). It is also well known that first integrals can be used to reduce the dimension of a dynamical system, but there are other alternatives that also make the reduction possible. One of them is the so called the reduction to the central manifold, that allows the simplification of the system, at least in the neighborhood of an equilibrium point. The center manifold of an equilibrium point, which is also called the slow manifold, has all the interesting dynamics around the point, since all the fast attracting and repelling motions, associated to the stable and unstable manifolds, are removed from the system in the reduction.

In this lecture we will talk about the application of the different kinds of invariant manifolds already mentioned in several problems in Astrodynamics, as well as some of the methodologies that can be used for their computation. We will see examples of its use to the computation of transfer orbits, the determination of practical stability regions, the definition of a station-keeping strategy along unstable orbits, the proof of the existence of orbits with prescribed itineraries, or the classification of the different kinds of libration point orbits.