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COMPUTING QUASI-PERIODIC TRAJECTORIES OF THE RTBP IN O(N LOG N) OPERATIONS

Abstract

Quasi-periodic (QP) trajectories play a fundamental role in many models in Astrodynamics, and provide many additional trajectory options in mission design beyond periodic motion [1,2]. They can be obtained locally by semi-analytical methods and globally by numerical ones. A numerical method that has progressively gained attention in the Astrodynamics community [1,2] is imposing invariance of a curve in the torus through the time-T flow (stroboscopic map), where T is one of the periods of the torus. Using a Fourier series approximation for the curve with N Fourier modes leads to a system of non-linear equations whose solution usually requires $O(N^3)$ operations. This is a computational bottleneck, especially for tori of more than two dimensions.

There is a new approach to the theoretical and computational study of invariant manifolds in dynamical systems known as the Parameterization Method [3,4,5]. In the computational side of this approach, differential correction is done in a functional framework, after the application of a transformation that makes use of geometrical properties that enable the computation of each Newton correction in a sequence of steps that are diagonal in either time or frequency domain. The computational benefit comes from the fact that the FFT allows to switch between these domains in $O(N \log N)$ operations.

In this paper, some theory and algorithms from [5] will be extended to enable the simultaneous computation of families of partially hyperbolic invariant tori together with the linear approximation of their stable and unstable manifolds. As a test, some of the families of tori of the RTBP (Lissajous, halo, quasi-halo) will be computed.

References

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