Paper ID: 41882
oral
student

45th STUDENT CONFERENCE (E2)
Student Conference - Part 2 (2)

Author: Mr. Langston Williams
Auburn University, United States

Prof. Joseph Majdalani
Auburn University, United States

DEVELOPMENT OF THE BIDIRECTIONAL VORTEX IN A HEMISPHERICALLY-SHAPED ROCKET ENGINE

Abstract

This study focuses on the development of an exact solution to the bidirectional vortex in a hemispherical chamber configuration. Our analysis proceeds from the Bragg-Hawthorne formulation, a variant of Euler's momentum equation that may be used in the treatment of steady, incompressible, and axisymmetric flowfields. This relation is used in conjunction with two fundamental assumptions that enforce a constant angular momentum and a variable stagnation enthalpy gradient. At the outset, a free vortex form is engendered in the chamber as well as an orthogonal relationship between the velocity vector and the gradient of the stagnation head. These physical requirements enable us to derive a closed-form expression for the bidirectional mean flow field in a hemispherical chamber while providing the flexibility to incorporate viscous corrections. Given the inviscid nature of the exact solution, viscous corrections are introduced to capture the inner core boundary layer that is needed to overcome the core singularity in the tangential velocity. After reducing the Bragg-Hawthorne equation, the ensuing expression is solved using a product solution of a squared sinusoidal expression and the spherical radial coordinate. The radially separable component is then simplified into a second-order differential equation that is capable of absorbing the problem's boundary requirements. In addition to the streamline boundary conditions that must be enforced along the wall, a balance between the inflow and the outflow is undertaken. This process enables us to consolidate our family of solutions into a compact form. With the mean flow streamfunction in hand, other properties of interest are derived, including the mantle location, which separates the outer annular motion from the inner, core flow. We find that the exit radius must be approximately seventy one percent of the chamber radius to coincide with the mantle location, and thus avoid collisions and recirculatory zones at entry. Along similar lines, the subsonic diffuser's inlet radius is determined from the mantle's location as the polar angle approaches ninety degrees.