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Author: Mr. Julian Brew  
Georgia Institute of Technology, United States, jbrew3@gatech.edu

Prof. Marcus Holzinger  
Georgia Institute of Technology, United States, holzinger@gatech.edu

USING REACHABILITY TO COMPUTE UNSAFE REGIONS IN STATE SPACE THROUGH  
SAMPLING METHODS

**Abstract**

There is an increased reliance on autonomous systems to assist in the operation of spacecraft and deep space operations. Additionally, there is the need of effective autonomous systems that adapt to unforeseen circumstances and uncertainties. The concept of reachability directly addresses these needs in system level autonomy. Given an initial set of states of a dynamic system and an understanding of the system dynamics, the set of reachable states after a certain amount of time - the forward reachability set - can be calculated or approximated. Conversely, given a final, target set of states and the dynamics of the system, the initial set of states that leads to the target set- the backward reachability set - can be computed. If the final, target set of states is considered unsafe, the backwards reachability set contains potentially unsafe state trajectories. Thus, backwards reachability analyses are used for safety assurance, fault detection, and collision avoidance applications.

Computing the reachability set for a dynamic system is derived from optimal control theory and it typically involves solving the Hamilton-Jacobi-Bellman partial differential equation using level-set techniques or polytopic over-approximations. Unfortunately, reachability computation suffers from the curse of dimensionality, and for large state spaces is computationally intractable. However, in many operational cases, the user is only interested in a subset of states in a reachability analysis (position states for collisions, for example); therefore, the user need only compute the subset reachability. As a result, an exponential improvement in computational cost is gained.

In this paper, a method of computing the backwards reachable set using continuation methods is described. Continuation methods allow for reachability optimal control boundary value problems to reduce to initial value problems that are solvable through numerical integration. Thus, point solutions that lie on the boundary of the reachability set can be computed by solving a collection of parallel initial value problems. This methodology has been implemented by Brew and Holzinger in forward reachability set analyses but not towards changing the methodology for backwards reachability set safety analyses.

The expected contributions from this research include: 1) The formulation of the optimal control policy and continuation method approach for backwards subspace reachability computations and 2) Demonstration of proposed approach for the scenario of computing the unsafe region in state space generated from an uncooperative spacecraft in proximity of a controlled spacecraft of interest.