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ANALYTICAL AND SEMI-ANALYTICAL APPROACHES TO THE THIRD-BODY PERTURBATION  
IN NEARLY CO-ORBITAL REGIMES**Abstract**

Since the beginning of space exploration, close encounters with celestial bodies in the Solar System have been exploited to change the motion of a spacecraft. Gravity assists are such an example; since they take place inside the planet's sphere of influence, their most used modelling framework is the patched-conics approximation. This, however, simplifies the spacecraft's motion as to be affected by only one celestial body at a time. Higher accuracy approaches, such as the circular-restricted three-body problem (CR3BP) models a simultaneous attraction of two bodies (primary and secondary: for example, Sun and Earth) and its application domain extends beyond the classical sphere of influence.

In between these approaches, perturbation techniques exist to account for the influence of the secondary, well outside the sphere of influence, in addition to the main attractive body. This paper presents two twin formulations for the variation of the spacecraft's orbital elements due to the third-body effect in the CR3BP, i.e. the regime of distant encounters outside the secondary's sphere of influence. These are based on the disturbing function of the previously studied Keplerian Map, derived from the Hamiltonian of the CR3BP in a barycentric coordinate system; additionally, they can be used in any kind of system of small gravitational parameter, such as the Sun-Earth one.

The first formulation is a partially analytical solution to the Lagrange planetary equations of motion. This strategy unites fully analytical equations for the evolution of the semi-major axis of the motion, which are obtained via a Taylor expansion on the eccentricity, with the use of the Euler method for the remaining unsolvable differential equations. This strategy allows the prediction of the orbital shape, making it potentially useful for fast online computations and application in GNC algorithms.

The second formulation is a mapping model for long time propagation, in which the orbital elements are updated at every periapsis and apoapsis to minimise the inherent numerical errors. This strategy is called the EK-PAP (Euler-Keplerian Periapsis to Apoapsis) map. It shows to be several orders of magnitude faster than the CR3BP, remaining accurate for motion durations up to several synodic periods. Particularly, the application to end-of-life disposal strategies is envisioned, in which the EK-PAP map can ensure that the long-term propagation of the disposed spacecraft follows the guidelines for clean space missions.