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ANALYTICAL SOLUTION OF LOW-THRUST MINIMUM ENERGY COPLANAR ORBIT TRANSFER BY USING TIME-AVERAGED CANONICAL EQUATIONS

Abstract

In this study, the analytical solution of the low-thrust minimum-energy orbit transfer problem is derived under the assumptions that the orbital motion is in the same plane throughout the transfer and that the transfer time is given. In order to solve the optimal control problem, the averaged canonical equations are derived by using the time-averaged Hamiltonian. The corresponding two boundary value problem is analytically integrated to form a set of nonlinear algebraic equations: One equation is expressed by hyperbolas and the other is by the inverse-tangent function. As the intersection point of two curves, the initial and the final values of the adjoint variables are analytically calculated. After solving the root of the nonlinear algebraic equations, the procedure to obtain the optimal trajectory and the osculating control input is presented. Moreover, the optimal value of the objective function is derived as a function of the initial orbital elements and is shown to be in the inverse-relationship to the total flight time. To show the validity of the proposed method, the orbital transfer of the geostationary spacecraft is calculated with different initial conditions (low earth orbit, medium earth orbit, geostationary transfer orbit, and the supersynchronous orbit). The proposed analytical solutions are compared with the numerical solutions by the pseudo-spectral method implemented by the MATLAB based optimal control software GPOPS-II, and are found to have good agreement for all the cases (the difference of the objective functions are less than 1 %). The analytical solution seems to be useful for the initial guess or the starting point of the homotopy-method for the other kinds of problem such as minimum-fuel or the minimum-time transfer problem.