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# ORBIT TRANSFER ANALYSIS WITH A 3D VIS-VIVA EQUATION

**Abstract**

This paper is a revision and extension of previous work considering ‘massdriver capabilities for bodies of the solar system’. Here a three dimensional version of the Vis-Viva equation has been derived that extends the scalar Vis-Viva equation allowing for a more generalized application for orbit analysis. This work is revised and extended to incorporate a full set of orbital elements and the implementation of relevant coordinate transformations to correctly analyse relative energies of generic starting and target orbits and obtain required velocity increments to perform spacecraft manoeuvres. When considering the kinetic and potential energy of a spacecraft in orbit as well as the respective velocity and position vectors a three dimensional energy vector can be calculated. This energy vector is oscillating along the orbit - as for example every quarter orbit the velocity vector direction and thus the energy vector turns by 90. Position vectors and velocity vector magnitudes for orbits of eccentricities unequal zero have non-linear influence. Applying known equations for a basic Hohmann transfer is only permitted for co-planar circular orbits, which is rarely the case for missions beyond Earth orbit. The 3D version of the Vis-Viva is a generalization of a Hohmann transfer reference case and is therefore capable to reproduce the same results. Hereby partial validation of the model is performed. Furthermore, utilizing a 3D Vis-Viva formulation is capable to produce Hohmann-like transfer orbits for more complex cases that considers orbits that are not co-planar and eccentric. For example, the interplanetary transfer from a low Earth orbit to a Mars orbit will consider four 3D energy vectors. The energy vector of the initial orbit around Earth, the energy vector of the Earth around the Sun, the energy vector of Mars around the Sun and the target vector in Mars orbit. As the 3D formulation of the Vis-Viva equation does consider the central body mass, the comparability and the ability for orbit analysis of these energy vectors is inherent. Considering the change of the energy vectors of full orbits and making analytical comparisons a minimal difference condition can be determined, which further yields required velocity increments of maneuvers as well as the position, where a transfer maneuver has to be performed.