

17th IAA SYMPOSIUM ON SPACE DEBRIS (A6)
Orbit Determination and Propagation (9)

Author: Mr. Shambo Bhattacharjee
University of Leeds, United Kingdom, mmsb@leeds.ac.uk

Prof. John T Kent
University of Leeds, United Kingdom, j.t.kent@leeds.ac.uk

Mr. Weston Faber
Applied Defense Solutions, Inc., United States, wfaber@applieddefense.com

Dr. Islam Hussein
Applied Defense Solutions, Inc., United States, ihussein@me.com

UNDERSTANDING THE DISTRIBUTION OF THE PROPAGATED ANGLES-ONLY POSITION
VECTOR**Abstract**

Consider a space object in an orbit about the earth with an initial Gaussian uncertainty for position and velocity in ECI coordinates. In this paper we explore the propagated uncertainty under Keplerian dynamics for the angles-only position vector. If the initial state defines an orbit in the equatorial plane, then the angles-only position vector can be described in terms of “latitude” and “longitude”, and various aspects of the distribution under propagation can be summarized as follows.

The distribution of longitude is approximately Gaussian for small propagation time. As the propagation time increases, the distribution becomes more spread out, eventually wrapping around the circle. It is then better described by the wrapped normal distribution, or (nearly equivalently) the von Mises distribution.

The conditional distribution of the latitude given longitude is more complicated to describe. It is conditionally Gaussian with mean 0, but the conditional variance depends on the longitude. We will give a complete description of the joint distribution of latitude and longitude using first order expansions in terms of the initial uncertainty, and we call this new distribution the “distorted Gaussian distribution”.

In many circumstances, the distorted Gaussian distribution simplifies to the bivariate Gaussian distribution (small propagation times), or to the previously developed Fisher-Bingham-Kent (FBK) distribution on the unit sphere (large propagation times).

However, in the case of a breakup event, where the initial position is known nearly exactly, but the initial velocity shows high uncertainty, the distorted Gaussian distribution shows a pronounced “pinching” or “bow-tie” effect in a scatterplot of latitude vs. longitude whenever the propagation time is an integer multiple of the half-period for the initial state. This behavior is well-known in breakup events where the cloud of debris tends to coalesce every half-integer period.

Implications of this new distorted Gaussian distribution for association problems will also be explored.