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TRANSFORMATION OF SATELLITE BREAK-UP DISTRIBUTION FUNCTION USING GAUSS'
VARIATIONAL EQUATIONS**Abstract**

To model the long-term evolution of the space debris environment, break-ups of satellites and upper stages need to be considered. Hundreds of explosions, and some collisions between space objects, have added thousands of observable objects and potentially millions of smaller fragments into the environment. Owing to the high orbital velocities, even small fragments can have detrimental effects on active space missions. Generally, the effects of fragmentations are estimated by sampling representative objects from a given break-up distribution function. The orbits of the objects are integrated over time, and the collision risk with a given mission of interest is calculated. Performing this process iteratively within a Monte Carlo simulation allows to draw statistical estimates of the space debris flux originating from fragmentation events. However, it is hard to infer the fragment density from representative objects, as the number of fragments that need to be propagated for a smooth derivation of the density is prohibitively large if the dimensionality, d , is larger than 3-4.

Another way of estimating the evolution of the distribution is to model the fragments as a continuum and integrate the continuity equation using the method of characteristics. Along each characteristic, an exact estimate of the fragment density is readily available, and the curse of dimensionality is mitigated. As long-term scenarios are of interest, the characteristics are semi-analytically integrated in Keplerian elements considering averaged dynamics. However, most break-up models, including the standard NASA break-up model, are formulated in cartesian coordinates, through a probability function with the delta- v as one of the dependent variables.

This paper proposes the application of Gauss' variational equation written for finite differences to transform a break-up distribution function given in cartesian coordinates into Keplerian elements. The first order variational equations allow to derive a linear transformation between the two frames. Given the transformation, the density distribution function can be converted in both directions. Singularities that lead to the reduction of the dimensionality are addressed. Limitations of the method for certain orbital configurations and large delta- v s are qualitatively discussed. Comparison against alternative methods to derive the density, such as binning, are performed to validate the method in low dimensional space (d smaller than 4). Binning, being a memory-intensive task, does not scale into many dimensions. The method is of importance as it facilitates the sampling of the initial characteristics to be integrated to estimate the evolution of one or multiple clouds of fragments.