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TRAJECTORY OPTIMIZATION USING MIXED MINIMUM-ERROR AND MINIMUM-FUEL COST FUNCTIONS

Abstract

An innovative approach to robust trajectory optimization will be discussed in which the final state error of a trajectory is minimized directly. Previous trajectory optimization methods have obtained robustness by minimizing the expected value of fuel use or by implementing reliability constraints to ensure that a fuel-optimal trajectory remains feasible under uncertainty. The approach taken in this paper will forgo fuel-optimality in order to find the most robust, minimum-error solution. Directly minimizing the error statistics of the final state may be useful for spacecraft orbiting in the sensitive microgravity environments around small bodies, where fuel requirements are low but uncertainty is large. Additionally, minimum-error optimization may be preferred for maneuvers where accuracy is of upmost importance, such as precision docking of crewed spacecraft. The minimum-error approach will also be extended to a dual minimum-error and minimum-fuel optimization through a weighted cost function. A dual-objective optimization will provide valuable insight into a trajectory design space and the trade-off between fuel-optimality and robustness. This analysis will consider two sources of uncertainty: initial state error (with covariance given by an uncertain navigation solution) and multiplicative control noise (for example, a thrust error that is proportional to the thrust magnitude). The minimum-error cost, J, will be a function of the expected value of the final state error, $\delta\mathbf{x}_{t_f}$, as seen in the following equation:

$$J = E[\delta\mathbf{x}_{t_f}^T \delta\mathbf{x}_{t_f}]. \tag{1}$$

The linearized dynamics of the state error with respect to a nominal trajectory can be described with the following stochastic differential equation:

$$d(\delta\mathbf{x}_t) = \frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u}, t)}{\partial \mathbf{x}} \delta\mathbf{x}_t dt + B\delta\mathbf{u}(t). \tag{2}$$

The control error $\delta\mathbf{u}$ will be modeled as a continuous Gaussian white noise signal. Ito calculus can be used to compute the expected value of the state deviation at the final time, and the cost function can be reformulated as

$$J = \mathrm{trace}\Big(\Phi(t_f, t_0) P_{\mathbf{x}_0\mathbf{x}_0} \Phi^T(t_f, t_0)\Big) + \int_{t_0}^{t_f} \mathrm{trace}\Big(\sigma_q^2 \Phi(\tau, t_f) \mathrm{B}\mathbf{u}(\tau)\mathbf{u}^T(\tau)\mathrm{B}^T \Phi^T(\tau, t_f)\Big) d\tau. \tag{3}$$

The minimum-error cost function takes the Bolza form and can be solved with traditional indirect optimal control methods. The minimum-error solution will be the open-loop continuous control law that minimizes the previous equation. Minimum-error optimization has been demonstrated for linear, time invariant systems (such orbit maneuvers in the Clohessy-Wiltshire equations) and nonlinear astrodynamics systems such as the two-body problem. Robust solutions will be compared to fuel-optimal solutions and verified through Monte Carlo simulations. It will be shown that minimum-error trajectories can differ significantly from fuel-optimal trajectories, and can provide significant improvements in robustness.