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# USING DIFFERENTIAL ALGEBRA TO COMPUTE LAGRANGIAN COHERENT STRUCTURES FOR MISSION DESIGN AND ANALYSIS

**Abstract**

Recent mission designs have exploited dynamical phenomena such as invariant manifolds to achieve fuel-efficient low-energy transfers. To aid the mission design process, methods have emerged in the literature to profile system dynamics and identify regions of different dynamical behaviour; orbits on the boundaries of these regions can often be used to design low-energy transfers between regions or points of interest in the system. In time-independent approximations to motion, such as the Circular-Restricted Three-Body Problem, the classical invariant manifolds partition phase space and separate dynamical behaviour, but in more realistic time-dependent models invariant manifolds generally do not exist. Lagrangian Coherent Structures (LCS) are a generalisation of the invariant manifold to dynamical systems with arbitrary time dependence and are defined with respect to the derivatives of the leading eigenvector of the deformation tensor for the flow, which provides a measure of how strongly nearby orbits diverge. Unfortunately, these derivatives can be numerically difficult and expensive to compute.

We present the results of DA-LCS, a new numerical method for the identification of three-dimensional LCS in time-dependent astrodynamical systems. We use Differential Algebra (DA) to automatically construct polynomial expansions of the flow with respect to its initial conditions, giving access to derivatives accurate to machine precision. A modified power law iteration is then used to construct similar polynomial expansions of the leading eigenvector of the deformation tensor as a function of the position. We demonstrate the effectiveness of DA-LCS compared to the traditional approximation of divided differences – which sometimes fails to produce usable insight despite altering the grid size used – and show the use of LCS in a series of example three-dimensional astrodynamical problems, which has thus far largely been limited to two-dimensional analyses. How LCS overcomes some of the false positives present in other methods, such as those based on the Finite-Time Lyapunov Exponent, is also elaborated. The structures obtained will be explained both in terms of the dynamical phenomena and how they can be used in the design of space missions, even when there is no *a priori* knowledge of system dynamics.