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Author: Mr. Max Hallgarten La Casta  
Imperial College London, United Kingdom

Dr. Davide Amato  
Imperial College London, United Kingdom  
Prof. Massimiliano Vasile  
University of Strathclyde, United Kingdom

POLYNOMIAL ALGEBRA FOR UNCERTAINTY PROPAGATION IN EQUINOCTIAL ORBITAL  
ELEMENTS

**Abstract**

The most straightforward way of propagating uncertainty distributions is through traditional Monte Carlo (MC) methods: a probability density function, usually assumed to be Gaussian, is sampled to produce a set of points which are propagated individually. The propagated states are used to build a final uncertainty distribution which is still assumed to be Gaussian. Due to the underlying dynamics, however, the Gaussian assumption will deviate from the true distribution. In this paper, a combination of Polynomial Algebra (PA) with Equinoctial Orbital Elements (EqOEs) is proposed as a solution with improvements over traditional MC-based Uncertainty Quantification (UQ) methods.

PA is a technique where values in the set of real numbers are replaced with polynomials, forming an algebra. The orbital dynamics can be expressed within the PA framework to propagate sets of states. PA provides a major advantage over traditional MC methods: uncertainties are handled as continuous distributions which are propagated only once. These continuous distributions can provide significantly more information than the discrete sample points from MC methods. Furthermore, computational performance is usually improved, particularly for cases with computationally expensive perturbation models. To date, propagation of uncertainty with PA has been restricted to using Cartesian coordinates as the dependent variables.

EqOEs are a set of orbital elements which provide several beneficial improvements over classical Keplerian elements, including the removal of singularities present for circular and equatorial orbits. These elements, including a recently introduced generalised version which embeds a perturbing potential in the elements' definition, have been explored for uncertainty propagation using linear methods. Under the influence of weak perturbations, EqOEs evolve slowly with time, unlike Cartesian states. Non-linearities in the dynamics evolve slowly in EqOEs, providing interesting characteristics when propagating uncertainty distributions. The uncertainty is easier to parametrise in EqOEs as variations will occur primarily in the single fast variable, in this case the mean longitude. Propagating uncertainties in EqOEs with PA enables increased fidelity of the uncertainty distribution, at a reduced computational cost.

In this paper, the theoretical setup for performing PA-based UQ in EqOEs is presented, including a method for converting from Cartesian to EqOEs avoiding discontinuities. The results from a numerical test campaign are presented, where propagations with PA using Cowell's method and EqOEs are compared to illustrate the relationship between the fidelity of the models, and computational time. The effect of different maximum degrees of the polynomials, geopotential perturbations, and sizes of initial uncertainty are also considered.