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# RESEARCH ON THE INTERPOLATION OF THE GRAVITATIONAL POTENTIAL OF THE EARTH

**Abstract**

Planning of Near-Earth space monitoring involves a large number of calculations related to forecasting the movement of space debris. A great part of the computational cost in this problem is due to determination of Earth gravitational force. Nowadays the representation of the geoid gravitational potential applying the partial sum of the Gaussian series is most commonly used:

$$U = \frac{\mu}{r}\left(1 + \sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{n} P_{nm}(sin\phi)\left[C_{nm}cos(m\lambda) + S_{nm}sin(m\lambda)\right]\right)$$

$(r, \phi, \lambda)$ - spherical coordinates of point, $\mu$ - gravitational parameter of the Earth, $P_{nm}$ - associated Legendre polynomials, $C_{nm}, S_{nm}$ - model coefficients. The calculation of a partial sum, for example, using the Clenshaw recursive sums algorithm, has a quadratic asymptotic complexity $O(N^2)$, where $N$ is a gravitational potential order. In addition, the determination of the potential or gravitational force contains a great number of operations due to the frequent calculation of trigonometric functions.

These circumstances have led the authors to the idea of different representation of the gravitational potential of the Earth and creation of an interpolant for the gravitational potential in near-Earth space. Constructing the interpolation once requires large computational resources. However, after its completion, the gravitational force can be calculated without significant costs: the calculation time does not rely on $N$ and does not require the computation of trigonometric functions. Two types of interpolation are considered: a tricubic spline and a three-dimensional interpolant on Chebyshev-Lissajous nodes.
In the case of a tricubic spline, the error in determining the gravitational force decreases cubically with a decrease in the size of the interpolation cell. To construct such a spline, it is necessary to calculate at each cell vertex the values of the gravitational potential, its first and second spatial partial derivatives, as well as a one-third mixed partial derivative. Testing has shown that in order to ensure the accuracy of determining the one-day trajectory of an object in near-Earth space of 10 meters, it is necessary to build a tricubic spline of 27 million cells.

Interpolation using Gauss-Lissajous nodes allows one to approximate the gravitational potential by a polynomial of almost arbitrary degree. To construct an interpolant, it is sufficient to calculate the value

of the function at certain points inside the interpolation cell. As a result of numerical experiments, it was found that the use of an interpolant of this type makes it possible to improve the accuracy of determining a one-day trajectory by 2 times in comparison with a tricubic spline with the same size of interpolation cells.

Both types of interpolation enable to reduce the time for calculating the gravitational force by about 1-2 orders of magnitude at $N = 16, .., 64$. The acceleration achieved in solving the equations of motion for such parameters is somewhat less and amounts to 5 - 10 times.