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REDUCING THE NUMERICAL ERROR OF ELEMENT-BASED FORMULATIONS FOR THE  
EVALUATION OF GRAVITY FIELDS**Abstract**

Traditionally, the modeling of non-uniform gravitational fields has relied on the classical expansions in solid spherical harmonics which, when it comes down to the practical realization of computer algorithms, has resulted over time in a variety of algorithms. Among the different possible classifications of these algorithms, a high-level breakdown divides them into coordinate-based and element-based methods.

Coordinate-based methods arise naturally from the fact that a gravity field is, by definition, a function of the position relative to the mass distribution generating the field. Therefore, the gravity field is conveniently expressed in a body-fixed frame, where the field remains invariant. Thus, the classical expansion in spherical harmonics is naturally derived in coordinate-based formulations. However, the increasing interest in general perturbation methods for the analytical propagation of orbits paved the way to a shift to element-based methods to evaluate the gravity field, since the latter enabled the direct evaluation of the gravity field as a function of the orbital elements themselves, avoiding recurring element-to-coordinate transformations, as in the well-known expansion of the gravity field in terms of the classical orbital elements, and the definition of Kaula's eccentricity and inclination functions.

The calculation of these functions, however, turned computationally heavy and cumbersome, so new algorithms were developed over time, which improved their numerical efficiency and stability. Interestingly though, when numerically computed, these functions yield a non-uniform error distribution across the values of their arguments. This is awkward, since it implies that the numerical accuracy of element-based methods is orbit dependant, thus violating the premise that the gravity field is a function of position only.

Interestingly, this a-priori undesirable effect can be rendered into an advantage, since the conversion between Cartesian coordinates and orbital elements is not unique if the velocity does not need to be matched, so it is possible to define not one, but infinitely many orbits that contain the specific location where the gravity field is to be evaluated. This yields a new degree of freedom that can be employed to ensure that Kaula's eccentricity and inclination functions are always evaluated at values of their arguments that minimize the numerical error, thus offering a suitable mechanism to mitigate the spurious numerical errors in the computation of element-based methods, and resulting in an improved accuracy of orbit propagations. This paper explores the effectiveness of this error mitigation mechanism, and quantifies its impact in orbital propagations based on simulated test scenarios.