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# BOUNDING NONLINEAR STRETCHING ABOUT SPACECRAFT TRAJECTORIES USING TENSOR EIGENPAIRS

#### Abstract

Linear approximations based on the linear term in the Taylor series are prevalent in spacecraft dynamics, guidance, navigation, and control because they enable analytical methods. However, linearization becomes inaccurate at large distances from the reference trajectory and/or when the underlying dynamical system is strongly nonlinear. When linear approximations are used for uncertainty propagation, they often underestimate uncertainty growth. Likewise, linearization-based methods like the finite time Lyapunov exponent (FTLE) can underestimate stretching in phase space. In this paper, we will present a semianalytical method to bound nonlinear stretching. We will demonstrate that the eigenpairs of higher-order tensors can be used to quantify the maximum stretching of a state deviation over time. The tenors in question are derived from the Taylor series expansion of a nonlinear dynamical system about a reference trajectory.

In our previous work, we developed a semianalytical measure of nonlinearity based on tensor eigenpairs. In the case of a dynamical system, these tensors are derived from the state transition tensor (STT) expansion. In this paper, we will apply a similar but novel approach to quantify the maximum stretching in phase space via tensor eigenpairs. It is well-known that the Cauchy-Green tensor (CGT) can be used to quantify the stretching of state deviations over time for a linear system: the maximum and minimum eigenvalues of the CGT determine the maximum and minimum amount that a state deviation may grow over some time interval. Moreover, the largest eigenvalue can be related to the FTLE. Because the CGT and the FTLE are based on the state transition matrix, they serve as an approximation for nonlinear systems. Alternatively, higher-order nonlinear analogs to the CGT and the FTLE will be presented in this paper to more accurately bound nonlinear stretching for applications in guidance, navigation, and control. Our analysis shows that higher-order terms are an important contribution to nonlinear stretching and uncertainty growth. Moreover, this methodology can provide more accurate descriptions of trajectory instabilities.