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VECTORIAL FORMULATION FOR THE PROPAGATION OF AVERAGE DYNAMICS UNDER GRAVITATIONAL EFFECTS

Abstract

A key part in the implementation of efficient orbit propagation codes is finding the more suitable formulation of the equations of motion. The separation of fast and slow effects is mostly desirable, yet not always possible, and is the primary goal of the variation of parameters method. For perturbed Keplerian motion, the "parameters" are different combinations of the fundamental integrals of the Kepler problem. To wit, the angular momentum vector and the eccentricity vector. Vectorial formulations avoid singularities and, although they commonly yield redundancy, the numerical integration is not harmed for the symmetry which normally arises from this kind of formulation. Moreover, since the bulk of the nonlinearities are effectively captured using mean elements, the formulation referred to the apsidal frame enjoys remarkable merits due to the effective isolation of the short-period effects.

In particular, the vectorial formulation of orbit propagation problems is commonly used in the long-term propagation of satellite orbits under third-body perturbations. In this case general formulas exist that provide the variation equations up to an arbitrary degree of the usual expansion of the third-body disturbing function. On the contrary, existing vectorial formulations for the case of geopotential perturbations are based on cumbersome hand computations, and, therefore, only include the effects of the lowest degree zonal harmonics. To avoid this limitation, we derive general expressions for the vectorial formulation of the equations of motion under geopotential perturbations up to an arbitrary number of zonal harmonics.

The dependence of the gravitational potential on the radius from the Earth's center of mass can be written in terms of its projections on the apsidal frame. In this way the fast components, related to the true anomaly, are clearly separated from the slow varying components, related to the unit vectors defining the slowly moving apsidal frame. Then, we combine the general representation of the Legendre polynomials in vectorial elements with that of the parallactic terms of the geopotential to obtain a general expression involving three finite summations for each zonal harmonic.

The new formulas naturally combine with analogous formulas modeling the third-body perturbations, and hence provide a methodical way of coding the efficient vectorial integration of circumterrestrial orbits during the long time spans that are mandatory in the preliminary design of mission orbits for artificial satellites. In addition, the new formulation is particularly useful in assessing the fidelity of different truncations of the gravitational potential in the propagation of particular orbits.