

IAF ASTRODYNAMICS SYMPOSIUM (C1)  
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Northwestern Polytechnical University, ChinaGENERALIZED GAUSSIAN SMOOTHING HOMOTOPY METHOD FOR SOLVING NONLINEAR  
OPTIMAL CONTROL PROBLEMS**Abstract**

Nonlinear optimal control problems are ubiquitous in the aerospace engineering field. To solve these problems, indirect methods, specifically addressing the two-point-boundary-value problem (TPBVP) derived from necessary conditions, have been extensively employed. These methods are well-known for their inherent accuracy and the potential to develop efficient algorithms for various applications. However, challenges associated with small convergence domains arise due to the sensitivity of solutions to unknown quantities. In the past few decades, significant efforts have been made to address these challenges, with homotopy methods emerging as a widely adopted approach.

Homotopy methods involve formulating a family of embedded problems parameterized by a homotopic parameter. Over the past few years, a variety of homotopy methods have been developed and successfully applied to challenging problems, such as low-thrust orbital transfer, planetary landing, orbital attitude control, and endo-atmospheric guidance. However, these methods are often specifically designed for distinct problems, resulting in a lack of generality. An exception is the recent advancement in smoothing homotopy methods, which incorporate the use of a smoothing kernel to convolve the state and costate variables in the time domain, and augment the differential equations with additional smoothed state and costate equations. However, these methods primarily apply convolution to the time domain and fail to fully resolve the sensitivity issue associated with the initial guess.

This paper proposes a new generalized smoothing Gaussian homotopy method to address the aforementioned issues. Unlike the original methods, the proposed approach extends convolution beyond the time domain to encompass all unknowns. A multivariate Gaussian function is utilized for the convolution of state and costate variables, a pivotal feature of which is the exploitation of the Gaussian kernel's separability property, enabling the decomposition of the multivariate Gaussian kernel into individual univariate Gaussian kernels along each dimension. This facilitates independent convolution operations in each dimension, significantly enhancing computational efficiency. In addition, Gauss-Chebyshev quadrature, a numerical integration method used to approximate definite integrals, is employed to calculate the univariate Gaussian convolutions associated with the initial costates, further reducing the computational load.

Challenging numerical examples are proposed, which demonstrate a significant enhancement in convergence compared to the original Gaussian smoothing homotopy method.