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EXPLOITING GAUGE FREEDOM IN KS VARIABLES FOR HIGH-PERFORMANCE NUMERICAL
ORBITAL PROPAGATION**Abstract**

Regularization has played a fundamental role in the development of modern Celestial Mechanics. The works of Kustaanheimo, Stiefel, Moser and Deprit among others paved the way into the modern sophistication of our understanding of the Kepler problem, described now in the language of differential geometry. Moreover, regularization was not only well-aligned with this mathematical reformulation, but also with the advent of computer-aided orbital propagation. In this sense, regularization offers significant advancements in terms of physical insight and enhanced accuracy within orbital propagation and navigation.

However, despite its long tradition, initiated by Euler, regularization is still a rich, active field of research, which shows both a plethora of available techniques and an equal number of physical and mathematical interpretations. Among others, the Kustaanheimo-Stiefel (KS) transformation remains as one of the most relevant. Aside its conceptual simplicity, it allows for efficient yet high-performance numerical routines. In short, the KS transformation, nowadays described in the language of quaternion algebra, renders the Keplerian problem into an isotropic harmonic oscillator, by means of a lifted Hopf map: the physical Cartesian space is elevated to an sphere in 4 dimensions.

Interestingly, the KS transformation is not injective: in the KS sphere, a locus of points (a Hopf fiber), generated by a gauge freedom, maps to the very same physical position. Moreover, the transformation can be understood to be generated by rotating a privileged direction, which is not a priori determined. Despite being physically irrelevant for the realization of orbital motion, these degrees of freedom have been shown to determine both the computational cost and accuracy of orbital propagation in KS variables.

By means of profiting from these gauge freedoms, this work presents a novel integration routine which effectively minimizes the propagation of numerical errors and alleviates the computational cost of performance-demanding orbital propagation. In particular, during integration time, this Hopf fiber, in combination with integrals of motion of the perturbed problem, is explored at will to enhance the accuracy of the propagation. Second, the above mitigation technique is combined with an online method to evolve the privileged generating direction of the KS transformation; again, with the same numerical objective. Third, we explore jumping between fibers for position-dependent only perturbed problems; seeking to exploit an invariance with respect to the orbiting body velocity, for increased integration accuracy.

Finally, the capabilities of the novel integrator are quantified based on simulated N -body and close-approaches orbital propagation scenarios.